Medical Image Segmentation Using Magnetostatic Active Contours (MAC) with Tensor Diffusion

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Abstract

In medical imagery, traditional deformable models often face substantial challenges due to fine structures and image complexity. Recently, based on magnetostatic theory, a new deformable model, namely MAC, is proposed for improving the ability of the active contour in dealing with complex geometries and segmentation difficulties. A Laplacian diffusion scheme is proposed in the MAC model to tackle excessive image noise which can interrupt image gradient vectors and in turn affect the external force field. In this paper, a derived vector potential field (VPF) is employed to obtain magnetic force and thus a diffusion tensor can be applied to diffuse VPF in terms of both magnitude and directional information, instead of directly diffusing the magnetic field as in the MAC model. Our diffusion is carried out both in spatial and temporal aspects of VPF so that the performance of the deformable model is significantly improved while images are with low signal-noise ratio (SNR) and poor contrast. In addition, the proposed diffusion enhancement can lead to evolving the curve smoothly and thus level set evolution is adapted to approach genuine object of interest. By applying in several medical image modalities, the results demonstrate the effectiveness of the proposed method.

1 Introduction

Due to natural biological variability and pathological conditions in medical imaging, various challenging problems arise in image segmentation, particularly low signal-noise ratio (SNR) and poor contrast. Over the decades, a substantial amount of methods have been developed to deal with these segmentation problems. Among many others, methods such as conventional active contour models, e.g. snake [6], gradient vector flow snake (GVF) [11] and geodesic active contours (GAC) [2], Markov random field (MRF) models [4], graph cuts [1], and piecewise constant models, e.g. Chan-Vese model [3], have been widely applied to medical image segmentation. It is generally perceived that region based approaches are more robust towards image noise and artefacts compared to edge based ones. However, region based approaches may suffer from inhomogeneity in regional characteristics, e.g. intensity or texture. Methods that have the properties of both approaches may offer a better solution in certain applications, particularly where object boundaries can not be simply described as discontinuities in intensity or regional characteristics.

Active contour model is a promising technique to medical image segmentation, due to its ability to handle complex object geometries. Numerous works have been reported in the literature. Explicit active contour models are usually represented by using parameterized splines such as snake [6], GVF [11], which are inadaptable to topological changes; in contrast, implicit active contour models are to embed the contour into a higher dimensional function so that these models can temporally adapt to contour propagation and topological changes. The level-set representation of the deforming contour has proved a powerful technique for numerically implementing the implicit active contour models such as GAC [2], the Chan-Vese model [3]. However, in medical imagery, these methods often face substantial challenges due to fine structures and image complexity, particularly convergence issues such as deep concavities, weak edges and broken boundaries. Recently, physics-inspired deformable models have been proposed for coping with these difficulties. For example, a charged particle model (CPM) [5] based on electrostatics was applied to localize object boundaries by assigning opposite charges to edge pixels and free particles. Another example is the elastic interaction based snake [9] which a long range interaction force based on the elastic interaction between line defects in solids is used as an external force in active contours. However, these methods still meet difficulties in dealing with the mentioned problems because weak edges can result in broken contours and noise can lead to curve evolution in wrong directions. Most recently, a new formulation for active contours based on magnetostatic field, called MAC, was introduced by Xie and Mirmehdi [10]. Instead of assigning fixed charges, MAC allows the charges flow through the edges and then a magnetic field is generated by the charge flow. Thus, the active contour is attracted towards the edges under the magnetic influence. Although the model is derived from simplistic edge based assumption, i.e. object boundary collocates with intensity discontinuity, the magnetic field computed from the interactions of those image gradient vectors behaves very similarly to a region based force. The MAC model shows significant improvements on the previous active contour models.

However, inevitably the MAC model will suffer from extensive image noise interference, which can disturb the gradient vectors and cause the inaccurate computation of the magnetic flux that will in turn affect curve evolution and lead to the deviation of the contour from genuine object boundaries. In [10], the authors proposed a Laplacian diffusion scheme to refine the magnetic field before contour evolution. In this paper, a derived vector potential field (VPF) is employed to obtain magnetic force and a diffusion tensor is then applied to diffuse VPF in terms of both magnitude and directional information. Thus, the diffusion is carried out both in spatial and temporal aspects of VPF so that the performance of MAC can significantly be improved for images with poor contrast and low SNR. In addition, the proposed diffusion enhancement can lead to evolving the curve smoothly and thus level set evolution is adapted to approach genuine object of interest. We apply the proposed method in two medical image modalities, angiography in eyes and urinary cast. The experimental results demonstrate the effectiveness of the proposed method. The remainder of this paper is organised as follows: In Section 2, a derived VPF is introduced and the calculation of magnetic flux with VPF is presented, and then tensor diffusion scheme for VPF is described. In Section 3, we present the experimental results in various medical image datasets. Finally, a conclusion is given in Section 4.

2 Method

2.1 Vector potential field

As described in [10], the direction of the currents, flows of charges, running through object boundary is estimated based on edge orientation, which is obtained by a 90° rotation in the image plane of the normalized image gradient vectors $(\hat{I}_x, \hat{I}_y)$, where I denotes an image. The image plane is considered as the X-Y plane in a 3D space Ω whose origin coincides with the origin of the image coordinates. Thus, the direction of object boundary current, $\mathbf{O}(\mathbf{x})$, is estimated as:

$$\mathbf{O}(\mathbf{x}) = (-1)^{\lambda}(-\hat{I}_y(\mathbf{x}), \hat{I}_x(\mathbf{x}), 0), \tag{1}$$

where $\mathbf{x}$ denotes a pixel position in the image domain, $\lambda = 1$ gives an anti-clockwise rotation in the image coordinates, and $\lambda = 2$ provides a clockwise rotation. In terms of the level set representation, the direction of current for the active contour, denoted as Υ, is similarly obtained by rotating the gradient vector $\nabla\Phi$ of the level set function Φ. Let $f(\mathbf{x})$ be the magnitude of image gradient, the magnetic flux $\mathbf{B}(\mathbf{x})$ generated by gradient vectors at each $\mathbf{x}$ is computed as:

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \sum_{\mathbf{s}\neq\mathbf{x}} f(\mathbf{s})\mathbf{O}(\mathbf{s}) \times \frac{\hat{\mathbf{R}}_{\mathbf{xs}}}{R_{\mathbf{xs}}^2}, \tag{2}$$

where μ_0 is the permeability constant, $\mathbf{s}$ denotes an edge pixel position, $\hat{\mathbf{R}}_{\mathbf{xs}}$ denotes a 3D unit vector from $\mathbf{x}$ to $\mathbf{s}$ in the image plane, and $R_{\mathbf{xs}}$ is the distance between them. The active contour is assigned with unit magnitude of electric current. The force imposed on it is derived as:

$$\mathbf{F}_m(\mathbf{x}) \propto \Upsilon(\mathbf{x}) \times \mathbf{B}(\mathbf{x}). \tag{3}$$

The magnetostatic active contour (MAC) model is then formulated as:

$$C_t = \alpha g(\mathbf{x})\kappa\hat{\mathbf{N}} + (1-\alpha)(\mathbf{F}_m(\mathbf{x}) \cdot \hat{\mathbf{N}})\hat{\mathbf{N}}, \tag{4}$$

where $g = 1/(1+f)$, κ denotes the curvature, and $\hat{\mathbf{N}}$ is inward unit normal. Note, $\mathbf{F}_m$ lies in the image domain and its third element equals zero, which can be ignored.

Furthermore, the magnetic flux density $\mathbf{B}$ shown in (2) can be described by its magnetic vector potential $\mathbf{A}(\mathbf{x})$:

$$\mathbf{B}(\mathbf{x}) = \nabla \times \mathbf{A}(\mathbf{x}), \mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \sum_{\mathbf{s}\neq\mathbf{x}} f(\mathbf{s}) \frac{\mathbf{O}(\mathbf{s})}{R_{\mathbf{xs}}}. \tag{5}$$

where $\mathbf{A}(\mathbf{x})$ can be expressed as $(A_i(\mathbf{x}), A_j(\mathbf{x}), 0)$ in Ω:

$$A_i(\mathbf{x}) = \frac{\mu_0}{4\pi} \sum_{\mathbf{s}\neq\mathbf{x}} f(\mathbf{s}) \frac{-\hat{I}_y(\mathbf{s})}{R_{\mathbf{xs}}}, \; A_j(\mathbf{x}) = \frac{\mu_0}{4\pi} \sum_{\mathbf{s}\neq\mathbf{x}} f(\mathbf{s}) \frac{\hat{I}_x(\mathbf{s})}{R_{\mathbf{xs}}}, \tag{6}$$

where we consider $\lambda = 1$ (see (1)). It does not make any theoretical difference if $\lambda = 2$ is used, which simply leads to $\mathbf{B}(\mathbf{x}, \lambda = 2) = -\mathbf{B}(\mathbf{x}, \lambda = 1)$. $(A_i, A_j, 0)$ is referred to as *vector potential field* or VPF for convenience. Equation (5) indicates that we can indirectly refine the magnetic field $\mathbf{B}$ by diffusing VPF, instead of post-processing $\mathbf{B}$ as proposed in [10] which, as we show in the experimental section, is problematic in noisy situations.

2.2 Tensor diffusion for vector potential field

For the vector potential field, referring to nonlinear diffusion techniques introduced in [8], we propose a tensor diffusion of the potential field before computing its circulation density. Thus, we diffuse the VPF by considering not only their magnitude but also their orientations

and then the magnetic flux $\mathbf{B}$ is calculated using the refined VPF according to (5). This will efficiently alleviate the disturbance of image noise or artefacts because the Laplacian diffusion used in [10] merely performs the scalar process in the magnetic flux $\mathbf{B}$. The following tensor diffusion scheme is employed for our purpose:

$$\frac{\partial}{\partial t}u - \nabla\cdot(\mathbf{D}(\nabla u)\nabla u) = \mathscr{F}(u_0), \tag{7}$$

where $u(t,\mathbf{x})$ is the diffused version, t can be considered as the "scale parameter", $\mathbf{D} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ is the diffusion tensor (a positive definite symmetric matrix), $\mathscr{F}$ can be considered as a penalty function which forces the diffusion result to conform to certain criteria, and $u_0(\mathbf{x}) = u(0,\mathbf{x})$ denotes the initial state. In our case, the VPF $\mathbf{A} = (A_i, A_j, 0)$ is a vector field. Thus, the nonlinear diffusion takes the following coupled form:

$$\begin{cases} \frac{\partial}{\partial t}\mathscr{A}_i - \nabla\cdot(\mathbf{D}(\nabla\mathscr{A}_i, \nabla\mathscr{A}_j)\nabla\mathscr{A}_i) = \mathscr{F}(A_i), \\ \frac{\partial}{\partial t}\mathscr{A}_j - \nabla\cdot(\mathbf{D}(\nabla\mathscr{A}_i, \nabla\mathscr{A}_j)\nabla\mathscr{A}_j) = \mathscr{F}(A_j), \end{cases} \tag{8}$$

where $\mathscr{A}_i(0,\mathbf{x}) = A_i(\mathbf{x})$, $\mathscr{A}_j(0,\mathbf{x}) = A_j(\mathbf{x})$. The diffusion tensor can be decomposed into two orthogonal components, one of which is parallel to the local potential vector and the other is perpendicular to the local vector. The orientation of a vector in the potential field can be denoted as $(\cos\theta, \sin\theta)$ and its orthogonal unit vector can be obtained as $(-\sin\theta, \cos\theta)$. Thus, (8) can be re-written as:

$$\frac{\partial}{\partial t}\mathscr{A} - \nabla\cdot\left(\mathbf{R}^T \begin{pmatrix} \omega & 0 \\ 0 & \gamma \end{pmatrix} \mathbf{R}\nabla\mathscr{A}\right) = \mathscr{F}(\mathbf{A}), \tag{9}$$

where $\mathbf{R} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, ω is the diffusion function in the direction of the VPF and γ denotes the diffusion function orthogonal to the field. Note the divergence and gradient operations are applied to each spatial component of $\mathscr{A}$ separately.

Considering the fact that we aim to have larger diffusion where potential vectors have smaller magnitude and preserve large potential vectors that are spatially consistent, we select the weighting function ω: $\omega(\mathbf{A}(\mathbf{x})) = e^{-\frac{|\mathbf{A}(\mathbf{x})|^3}{K'}}$, where K' is the parameter controlling the amount of diffusion ($K' = 0.25$ in our experiments) and $|\mathbf{A}| \in [0,1]$. In addition, the diffusion perpendicular to local potential vectors plays a critical role in propagating the potential vectors from strong edges to regions further away from them, which may be dominated by image noise. We wish to increase the diffusion in this perpendicular direction according to the magnitude of the VPF, thus we choose the square root of the VPF magnitude, $|\mathbf{A}|^{0.5}$, as a measure of degradation along this perpendicular direction and then similarly define the diffusion function $\gamma = e^{|\mathbf{A}|^{0.5}}$.

For obtaining the solution with a nontrivial steady state in (9) and avoiding the problem of choosing a stopping time [7], we define the conformity function $\mathscr{F}$ as $\mathscr{F}(\mathbf{A}(\mathbf{x})) = \mathbf{A}(\mathbf{x}) - \mathscr{A}(\mathbf{x})$. Moreover, a weight is used to exert substantial diffusion for desirable regions, i.e. the larger ω is, the less constraint is imposed on conformity. Thus, $\mathscr{F}$ is given as:

$$\mathscr{F}(\mathbf{A}(\mathbf{x})) = (1 - \omega(\mathbf{A}(\mathbf{x})))(\mathbf{A}(\mathbf{x}) - \mathscr{A}(\mathbf{x})). \tag{10}$$

3 Experimental results

The proposed diffusion method has been applied in refining the magnetic flux of the MAC model for extracting the ROIs in medical images. The controlling parameter K' is set to

0.25 in the experiments. We compare this indirect diffusion method with the Laplacian diffusion method used in [10]. The first image modality used is from the angiography in eyes, which is usually with low SNR and poor contrast. The initialisations of two cases are presented in Fig. 1(a)(d). The results in Fig. 1(c)(f) illustrate that our method can extract more accurate boundaries of the vessels in comparison to the Laplacian method (Fig. 1(b)(e), see the arrowed places). Fig. 2 illustrates the curve evolution of an example using our method and the Laplacian. Three more examples in Fig. 3 show the superior performance of the proposed diffusion method in comparison with the Laplacian method. We can see that the acquired boundaries using the proposed method are not only more accurate, but also more smoother than using the Laplacian method. This is due to performing the tensor diffusion in terms of both the magnitude and orientation information of VPF.

Furthermore, we have applied our method to another medical image modality, urinary cast, which are often with low SNR and poor contrast as well. Four examples are presented in Fig. 4 where the largest components are the interested cellular structures in the images. The initialisations are shown in Fig. 4(a1)(b1)(c1)(d1). Fig. 4(a2)(b2)(c2)(d2) show the results using the Laplacian diffusion and Fig. 4(a3)(b3)(c3)(d3) present the results using our method. In this difficult situation, the proposed diffusion method can acquire accurate smooth boundaries while the Laplacian diffusion works by contrast ineffective.

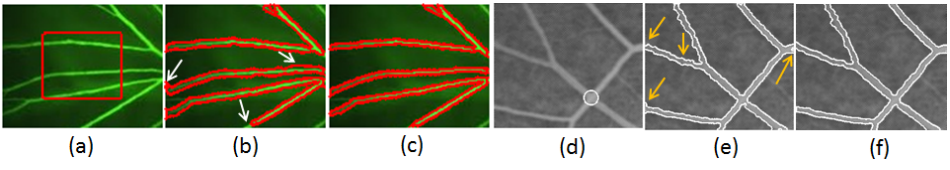

Figure 1: Segmentation results for two angiography images. (a,d) are the initialisations, (b,e) are the results using the Laplacian diffusion. (c,f) are the results using the proposed method. The arrowed places are problematic.

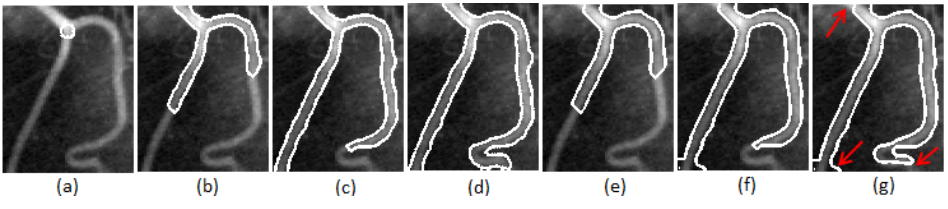

Figure 2: Curve evolution using our method. (a) is the initialisation; (b,e), (c,f) and (d,g) are the intermediate results for iteration 40, iteration 90 and the final one using our method and the Laplacian respectively. The arrowed places are problematic

4 Conclusions

In this paper, we propose a tensor diffusion method to diffuse the derived VPF so that the magnetic flux **B** can be refined better than the Laplacian method used in [10]. The major flavor of the proposed diffusion method is due to considering both diffusion magnitude and orientations in the diffusion process that can significantly improve the segmentation performance of the MAC model, particular in images with low SNR and poor contrast. We apply the proposed method in several medical image modalities. The current experimental results illustrate its superior performance in comparison to the Laplacian diffusion method. Future work will focus on the performance evaluation of the proposed method.

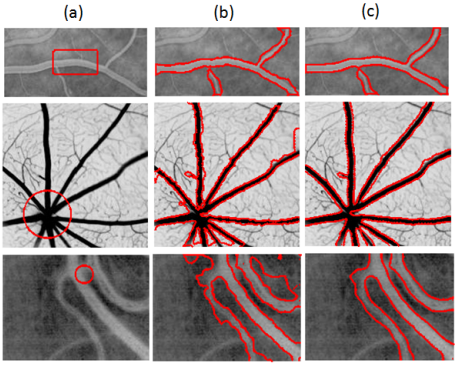

Figure 3: Extra examples for angiography. Column (a) is the initialisation, Column (b) is the results using the Laplacian method and Column (c) is the results using the proposed method.

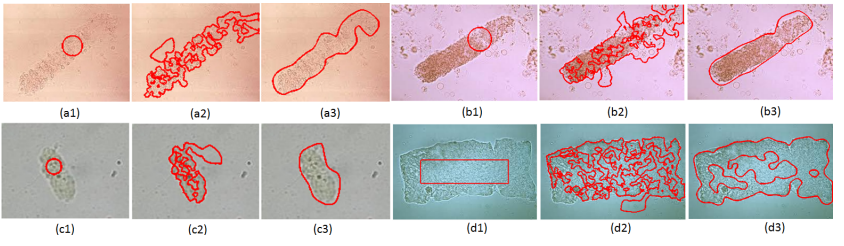

Figure 4: Results for urinary cast. (a1,b1,c1,d1) are the initialisations, (a2,b2,c2,d2) are the results using Laplacian diffusion, (a3,b3,c3,d3) are the results using the proposed method.

References

[1] Y. Boykov and G. Funka-Lea. Graph cuts and efficient n-d image segmentation. *IJCV*, 70(2): 109–131, 2006.

[2] V. Caselles, R. Kimmel, and G. Sapiro. Geodesic active contour. *IJCV*, 22(1):61–79, 1997.

[3] T. Chan and L. Vese. Active contours without edges. *T-IP*, 10(2):266–277, 2001.

[4] S. Geman and D. Geman. Stochastic relaxation, Gibbs distribution and the Bayesian restoration of images. *IEEE. Tran. Pattern Anal. Mach. Intell.*., 6(6):721–741, 1984.

[5] A. Jalba, M. Wilkinson, and J. Roerdink. CPM: A deformable model for shape recovery and segmentation based on charged particles. *T-PAMI*, 26:1320–1335, 2004.

[6] M. Kass, A. Witkin, and D. Terzopoulus. Snakes: Active contour model. *IJCV*, 1(4):321–331, 1988.

[7] K. Nordström. Biased anisotropic diffusion - a unified regularization and diffusion approach to edge detection. In *ECCV*, pages 18–27, 1990.

[8] J. Weickert. *Anisotropic Diffusion in Image Processing*. Teubner-Verlag, Stuttgart, Germany, 1998.

[9] Y. Xiang, A. Chung, and J. Ye. An active contour model for image segmentation based on elastic interaction. *J. Comp. Phy.*, 219(1):455–476, 2006.

[10] X. Xie and M. Mirmehdi. MAC: Magnetostatic active contour. *T-PAMI*, 30(4):632–646, 2008.

[11] C. Xu and J. Prince. Snakes, shapes, & gradient vector flow. *IP*, 7(3):359–369, 1998.